OCR
Oxford Cambridge and RSA

Tuesday 19 October 2021 – Afternoon

AS Level Further Mathematics A

Y533/01 Mechanics

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 One end of a light inextensible string of length 2.8 m is attached to a fixed point O on a smooth horizontal table. The other end of the string is attached to a particle P which moves on the table, with the string taut, in a circular path around O. The speed of P is constant and P completes each circle in 0.84 seconds.

(a) Find the magnitude of the angular velocity of P. **[2]**

(b) Find the speed of P. **[2]**

(c) Find the magnitude of the acceleration of P. **[2]**

(d) State the direction of the acceleration of P. **[1]**

2 A car has a mass of 800 kg. The engine of the car is working at a constant power of 15 kW.

In an initial model of the motion of the car it is assumed that the car is subject to a constant resistive force of magnitude R N.

The car is initially driven on a straight horizontal road. At the instant that its speed is $20\,\mathrm{ms}^{-1}$ its acceleration is $0.4\,\mathrm{ms}^{-2}$.

(a) Show that $R = 430$. **[3]**

(b) Hence find the maximum constant speed at which the car can be driven along this road, according to the initial model. **[2]**

In a revised model the resistance to the motion of the car at any instant is assumed to be $60v$ where v is the speed of the car at that instant.

The car is now driven up a straight road which is inclined at an angle α above the horizontal where $\sin\alpha = 0.2$.

(c) Determine the speed of the car at the instant that its acceleration is $0.15\,\mathrm{ms}^{-2}$ up the slope, according to the revised model. **[4]**

3 A particle A of mass 0.5 kg is moving with a speed of $3.15\,\mathrm{m\,s^{-1}}$ on a smooth horizontal surface when it collides directly with a particle B of mass 0.8 kg which is at rest on the surface. The velocities of A and B immediately after the collision are denoted by $v_A\,\mathrm{m\,s^{-1}}$ and $v_B\,\mathrm{m\,s^{-1}}$ respectively. You are given that $v_B = 2v_A$.

(a) Find the values of v_A and v_B. **[3]**

(b) Find the coefficient of restitution between A and B. **[2]**

(c) Explain why the coefficient of restitution is a dimensionless quantity. **[1]**

(d) Calculate the total loss of kinetic energy as a result of the collision. **[3]**

(e) State, giving a reason, whether or not the collision is perfectly elastic. **[1]**

(f) Calculate the impulse that B exerts on A in the collision. **[3]**

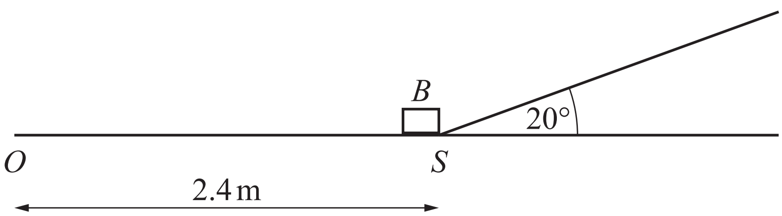
4 A small box B of mass 4.2 kg is initially at rest at a point O on rough horizontal ground. A horizontal force of magnitude 35 N is applied to B.

B moves in a straight line until it reaches the point S which is 2.4 m from O. At the instant that B reaches S its speed is $4.5\,\mathrm{m\,s^{-1}}$.

(a) (i) Find the energy lost due to the resistive forces acting on B as it moves from O to S. **[3]**

(ii) Deduce the magnitude of the average resistive force acting on B as it moves from O to S. **[2]**

When B reaches S, the force is no longer applied. B continues to move directly up a smooth slope which is inclined at 20° above the horizontal (see diagram).



(b) (i) State an assumption required to model the motion of B up the slope with only the information given. **[1]**

(ii) Using the assumption made in part **(b)(i)**, determine the distance travelled by B up the slope until the instant when it comes to rest. **[3]**

5 The escape speed of an **unpowered** object is the minimum speed at which it must be projected to escape the gravitational influence of the Earth if it is projected vertically upwards from the Earth's surface. A formula for the escape speed U of an unpowered object of mass m is $U = \sqrt{\frac{2Gm}{r}}$ where r is the radius of the Earth and G is a constant.

(a) Show that the dimensions of G are $\mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2}$. **[3]**

A rocket is a **powered** object. A rocket is launched with a given **launch speed** and is then powered by engines which apply a constant force for a period of time after the launch.

A student wishes to apply the formula given above to a rocket launch. They wish to model the minimum launch speed required for a rocket to escape the Earth's gravitational influence.

They realise that the given formula is for unpowered objects and so they include an extra term in the formula to obtain $V = \sqrt{\frac{2Gm}{r}} - kP^{\alpha}W^{\beta}t^{\gamma}$.

In their modified formula, G and r are the same as before. The other variables are defined as follows.

- V is the required minimum launch speed of the rocket
- k, α, β and γ are dimensionless constants
- P is the power developed by the engines of the rocket
- m is the initial mass of the rocket
- W is the initial weight of the rocket
- t is the total time for which the engines of the rocket operate

(b) Use dimensional analysis to determine the values of α, β and γ. **[6]**

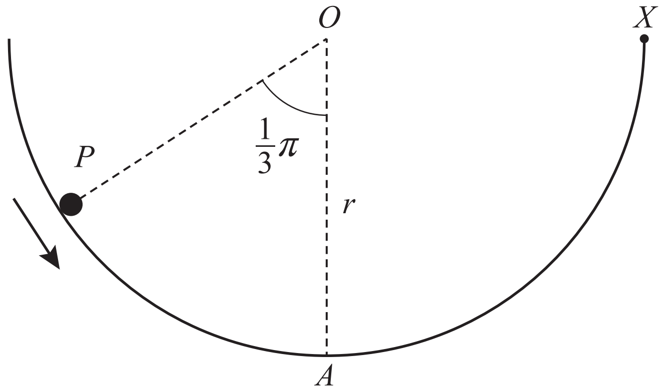
(c) By considering the value of γ found in part **(b)** explain the relationship between t and V. **[1]**

6 A smooth hemispherical shell of radius r m is held with its circular rim horizontal and uppermost. The centre of the rim is at the point O and the point on the inner surface directly below O is A.

A small object P of mass m kg is held at rest on the inner surface of the shell so that $\angle POA = \frac{1}{3}\pi$ radians. At the instant that P is released, an impulse is applied to P in the direction of the tangent to the surface at P in the vertical plane containing O, A and P. The magnitude of the impulse is denoted by I Ns.

P immediately starts to move along the surface towards A (see diagram).

X is a point on the circular rim. P leaves the shell at X.



In an initial model of the motion of P it is assumed that P experiences no resistance to its motion.

(a) Find in terms of r, g, m and I an expression for the speed of P at the instant that it leaves the shell at X. **[4]**

(b) Find in terms of r, g, m and I an expression for the maximum height attained by P above X after it has left the shell. **[1]**

(c) Find an expression for the maximum mass of P for which P still leaves the shell. **[2]**

In a revised model it is assumed that P experiences a resistive force of constant magnitude R while it is moving.

(d) Show that, in order for P to **still** leave the shell at X under the revised model,

$$I > \sqrt{m^2gr + \frac{5\pi mrR}{3}}.$$ **[3]**

(e) Show that the inequality from part **(d)** is dimensionally consistent. **[2]**

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